

# Asymptotic Normality of the Additive Regression Components for Continuous Time Processes

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## ABSTRACT

In multivariate regression estimation, the rate of convergence depends on the dimension of the regressor. This fact, known as the *curse of the dimensionality*, motivated several works. The additive model, introduced by Stone (10), offers an efficient response to this problem. In the setting of continuous time processes, using the marginal integration method, we obtain the quadratic convergence rate and the asymptotic normality of the components of the additive model.

## 1 Introduction

Let  $\mathbf{Z}_t = (\mathbf{X}_t, Y_t)_{(t \in \mathbb{R})}$  be a  $\mathbb{R}^d \times \mathbb{R}$ -valued measurable stochastic process defined on a probability space  $(\Omega, \mathcal{A}, P)$  with  $d \geq 1$ . Let  $\mathcal{C}_1, \dots, \mathcal{C}_d$ , be  $d$  compact intervals of  $\mathbb{R}$  and set  $\mathcal{C} = \mathcal{C}_1 \times \dots \times \mathcal{C}_d$ . Set now  $\delta > 0$  and introduce the  $\delta$ -neighborhood  $\mathcal{C}^\delta$  of  $\mathcal{C}$ , namely  $\mathcal{C}^\delta = \{\mathbf{x} : \inf_{\mathbf{y} \in \mathcal{C}} \|\mathbf{x} - \mathbf{y}\|_{\mathbb{R}^d} < \delta\}$ , with  $\|\cdot\|_{\mathbb{R}^d}$  standing for the euclidian norm on  $\mathbb{R}^d$ . Let  $\psi$  be a real valued measurable function. Consider the regression function  $m_\psi$  defined by,

$$m_\psi(\mathbf{x}) = E(\psi(Y) | \mathbf{X} = \mathbf{x}), \quad \forall \mathbf{x} = (x_1, \dots, x_d) \in \mathcal{C}^\delta. \quad (1)$$

Let  $K$  be a kernel defined on  $\mathbb{R}^d$  and having a compact support. Let  $\hat{f}_T$  be the estimate of  $f$ , the density function of the covariable  $\mathbf{X}$ , (see Banon (1)), defined by,

$$\hat{f}_T(\mathbf{x}) = \frac{1}{Th_T^d} \int_0^T K\left(\frac{\mathbf{x} - \mathbf{X}_s}{h_T}\right) ds,$$

where  $h_T$  is a given real positive function. In the sequel, to estimate the regression function defined in (1), we use the following estimator (see, for example, Bosq (3) and Jones et al.(7))

$$\tilde{m}_{\psi,T}(\mathbf{x}) = \int_0^T W_{T,t}(\mathbf{x}) \psi(\mathbf{Y}_t) dt \quad \text{with} \quad W_{T,t}(\mathbf{x}) = \frac{\prod_{l=1}^d \frac{1}{h_{l,T}} K_l\left(\frac{\mathbf{x}_t - \mathbf{X}_t}{h_{l,T}}\right)}{T \hat{f}_T(\mathbf{X}_t)}, \quad (2)$$

where  $(h_{j,T})_{1 \leq j \leq d}$  are positive real functions and  $(K_l)_{1 \leq j \leq d}$  are  $d$  kernels defined on  $\mathbb{R}$  with compact supports. Consider now that the nonparametric regression function (1) may be written as a sum of univariate functions, i.e.

$$m_\psi(\mathbf{x}) \equiv \mu + \sum_{l=1}^d m_l(x_l) =: m_{\psi,add}(\mathbf{x}), \quad \forall \mathbf{x} = (x_1, \dots, x_d) \in \mathcal{C}^\delta, \quad (3)$$

where, for  $1 \leq l \leq d$ ,  $Em_l(X_l) = 0$ . For  $1 \leq l \leq d$  and any  $\mathbf{x} = (x_1, \dots, x_d) \in \mathcal{C}^\delta$  set  $\mathbf{x}_{-l} = (x_1, \dots, x_{l-1}, x_{l+1}, \dots, x_d)$ . To estimate the additive components, we use the marginal integration method (see Linton & Nielsen (8) and Newey (9)). To this aim, we introduce  $d$  densities  $q_1, \dots, q_d$  defined on  $\mathbb{R}$  and set  $q(\mathbf{x}) = \prod_{l=1}^d q_l(x_l)$  and  $q_{-l}(\mathbf{x}_{-l}) = \prod_{j \neq l} q_j(x_j)$   $\{l = 1, \dots, d\}$ . We can then write

$$m_\psi(\mathbf{x}) = \sum_{l=1}^d \eta_l(x_l) + \int_{\mathbb{R}^d} m_\psi(\mathbf{z}) q(\mathbf{z}) d\mathbf{z} \quad (4)$$

with

$$\begin{aligned} \text{with } \eta_l(x_l) &:= \int_{\mathbb{R}^{d-1}} m_\psi(\mathbf{x}) q_{-l}(\mathbf{x}_{-l}) d\mathbf{x}_{-l} - \int_{\mathbb{R}^d} m_\psi(\mathbf{x}) q(\mathbf{x}) d\mathbf{x} \\ &= m_l(x_l) - \int_{\mathbb{R}} m_l(z) q_l(z) dz, \quad 1 \leq l \leq d. \end{aligned} \quad (5)$$

Making use of the statements (2) and (5), it follows that a natural estimate of the  $l$ -th component is given by

$$\hat{\eta}_{l,T}(x_l) = \int_{\mathbb{R}^{d-1}} \tilde{m}_{\psi,T}(\mathbf{x}) q_{-l}(\mathbf{x}_{-l}) d\mathbf{x}_{-l} - \int_{\mathbb{R}^d} \tilde{m}_{\psi,T}(\mathbf{x}) q(\mathbf{x}) d\mathbf{x}, \quad l = 1, \dots, d. \quad (6)$$

## 2 Hypotheses and Notations

In order to state our results, we introduce some assumptions and additional notations.

(C.1) There exists a positive constant  $M$  such that, for any  $y \in \mathbb{R}$ ,  $|\psi(y)| \leq M < \infty$ ,

(C.2)  $m_\psi$  is a  $k$ -times continuously differentiable function,  $k \geq 1$ , and

$$\sup_{\mathbf{x}} \left| \frac{\partial^k m_\psi}{\partial x_l^k}(\mathbf{x}) \right| < \infty; \quad 1 \leq l \leq d.$$

For  $1 \leq l \leq d$ , we denote by  $f_l$ , the density function of  $X_l$  and we suppose that the functions  $f$  and  $f_l$  are continuous and bounded. We need the additional conditions

(F.1)  $\forall \mathbf{x} \in \mathcal{C}^\delta$ ,  $f(\mathbf{x}) > 0$  and  $f_l(x_l) > 0$ ,  $l = 1, \dots, d$ ,

(F.2)  $f$  is  $k'$ -times continuously differentiable on  $\mathcal{C}^\delta$ ,  $k' > kd$ ,

(F.3) for some  $0 < \lambda \leq 1$ ,  $\left| \frac{\partial f^{(k')}}{\partial x_1^{j_1} \dots \partial x_d^{j_d}}(\mathbf{x}') - \frac{\partial f^{(k')}}{\partial x_1^{j_1} \dots \partial x_d^{j_d}}(\mathbf{x}) \right| \leq L \|\mathbf{x}' - \mathbf{x}\|^\lambda$  with  $j_1 + \dots + j_d = k'$ .

Here  $\|\cdot\|$  states as a norm on  $\mathbb{R}^d$ ,  $L$  is a positive constant and we note  $r := k' + \lambda$ .

The kernels  $K$  and  $K_l, 1 \leq l \leq d$  are assumed to fulfill the following conditions

- (K.1) For  $1 \leq l \leq d$ ,  $K$  and  $K_l$  are continuous on compact supports  $S$  and  $S_l \subset \mathcal{C}_l$ , respectively,
- (K.2)  $\int K = 1$  and  $\int K_j = 1, \quad 1 \leq l \leq d$ ,
- (K.3)  $\prod_{j=1}^d K_j$  is of order  $k$ ,
- (K.4)  $K$  is of order  $k'$ .

The known integration density functions  $q_l, 1 \leq l \leq d$ , satisfy the following assumption

- (Q.1)  $q_l$  has  $k$  continuous and bounded derivatives, with compact support included in  $\mathcal{C}_l, \quad 1 \leq l \leq d$ .

There exists  $\Gamma \in \mathcal{B}_{\mathbb{R}^2}$  containing  $D = \{(s, t) \in \mathbb{R}^2 : s = t\}$  such that

- (D.1)  $f_{(\mathbf{x}_s, Y_s), (\mathbf{x}_t, Y_t)} - f_{(\mathbf{x}_s, Y_s)} \otimes f_{(\mathbf{x}_t, Y_t)}$  exists everywhere for  $(s, t) \in \Gamma^C$ ,
- (D.2)  $A_\Gamma := \sup_{(s, t) \in \Gamma^C} \sup_{\mathbf{x}, \mathbf{y} \in \mathcal{C}^\delta \times \mathcal{C}^\delta} \int_{u, v \in \mathbb{R}^2} |f_{(\mathbf{x}_s, Y_s), (\mathbf{x}_t, Y_t)}(\mathbf{x}, u, \mathbf{y}, v) - f_{(\mathbf{x}_s, Y_s)}(\mathbf{x}, u) f_{(\mathbf{x}_t, Y_t)}(\mathbf{y}, v)| du dv < \infty$ ,
- (D.3) there exists  $\ell_\Gamma < \infty$  and  $T_0$  such that,  $\forall T > T_0, \quad \frac{1}{T} \int_{[0, T]^2 \cap \Gamma} ds dt \leq \ell_\Gamma$ .

We will work under the following conditions on the smoothing parameters  $h_T$  and  $h_{j,T}, j = 1, \dots, d$ .

- (H.1)  $h_T = c' \left( \frac{\log T}{T} \right)^{1/(2k'+d)}$ , for a fixed  $0 < c' < \infty$ ,
- (H.2)  $h_{j,T} = c_1 T^{-1/(2k+1)}$ , for fixed  $0 < c_1 < \infty$ .

Let  $\mathcal{A}$  and  $\mathcal{B}$  be two  $\sigma$ -fields. We will use the  $\alpha$ -mixing coefficient defined by

$$\alpha(\mathcal{A}, \mathcal{B}) = \sup_{(A, B) \in (\mathcal{A}, \mathcal{B})} |P(A \cap B) - P(A)P(B)|.$$

For all Borel set  $I \subset \mathbb{R}^+$  the  $\sigma$ -algebra defined by  $(Z_t, t \in I)$  will be denoted by  $\sigma(Z_t, t \in I)$ . Writing  $\alpha(u) = \sup_{t \in \mathbb{R}_+} \alpha(\sigma(Z_v, v \leq t), \sigma(Z_v, v \geq t + u))$ , we will use the condition

- (A.1)  $\alpha(t) = \mathcal{O}(t^{-b})$  with  $b > \frac{7r+5d}{2r}$ .

We denote by  $\widehat{\eta}_{l,T}$  and  $\widetilde{m}_{\psi,T}(\mathbf{x})$  the versions of  $\widehat{\eta}_{l,T}$  and  $\widetilde{m}_{\psi,T}(\mathbf{x})$  corresponding to a known density  $f$ . Introduce now the following quantities (see, for the discrete case, Camlong *et al.* (4)),

$$\begin{aligned}
\tilde{Y}_{\psi,T,t,l} &= \psi(Y_t) \int_{\mathbb{R}^{d-1}} \prod_{j \neq l}^d \frac{1}{h_{j,T}} K_j \left( \frac{\mathbf{x}_j - \mathbf{X}_{t,j}}{h_{j,T}} \right) \frac{q_{-l}(\mathbf{x}_{-l})}{f(X_{t,-l}|X_{t,l})} d\mathbf{x}_{-l}; \tilde{m}_{\psi,l}^T(x_l) = E(\tilde{Y}_{\psi,T,t,l} | X_{t,l} = x_l); \\
\hat{\alpha}_l(x_l) &= \frac{1}{Th_{l,T}} \int_0^T \frac{\tilde{Y}_{\psi,T,t}}{f_1(X_{t,l})} K_l \left( \frac{x_l - X_{t,l}}{h_{l,T}} \right) dt; \mathcal{G}_l(\mathbf{u}_{-l}) = \int_{\mathbb{R}^{d-1}} \prod_{j \neq l}^d \frac{1}{h_{j,T}} K_j \left( \frac{\mathbf{x}_j - \mathbf{u}_j}{h_{j,T}} \right) q_{-l}(\mathbf{x}_{-l}) d\mathbf{x}_{-l}; \\
C_{T,l} &= \mu + \int_{\mathbb{R}^{d-1}} \sum_{j \neq l} m_j(u_j) \mathcal{G}_l(\mathbf{u}_{-l}) d\mathbf{u}_{-l}; \hat{C}_T = \int_{\mathbb{R}^d} \tilde{m}_{\psi,T}(\mathbf{x}) q(\mathbf{x}) d\mathbf{x}; C_l = \int_{\mathbb{R}} m_l(x_l) q_l(x_l) dx_l; \\
b_l(x_l) &= \frac{1}{k!} \int_{\mathbb{R}} u^k K_l(u) du \left( (-1)^k m_l^{(k)}(x_l) + \int_{\mathbb{R}} m_l(z) q_l^{(k)}(z) dz \right).
\end{aligned}$$

### 3 Results

The proofs of our Theorems are split into two steps. We first consider the density as known, and then treat the general case where  $f$  is unknown by using the decomposition  $1/f = 1/\hat{f}_T - (f - \hat{f}_T)/f\hat{f}_T$  and the following lemma.

**Lemma 1** *Under the assumptions (F.1) – (F.3), (K.1), (K.2), (K.4), (D.1) – (D.3), (H.1) and (A.1) we have*

$$\sup_{\mathbf{x} \in \mathcal{C}} |\hat{f}_T(\mathbf{x}) - f(\mathbf{x})| = \mathcal{O} \left( \left( \frac{\log T}{T} \right)^{k'/(2k'+d)} \right) \quad a.s.. \quad (7)$$

**Proof:** It is easily seen that under our assumptions, the result follows by using the arguments used in the demonstration of Theorem 4.9. in (2) p.112 and by replacing  $\log_m$  by 1.

**Theorem 1** *Under assumptions (C.1) – (C.2), (F.1) – (F.3), (K.1) – (K.4), (Q.1), (D.1) – (D.3), (H.1) – (H.2) and (A.1) we have*

$$E(\hat{\eta}_{l,T}(x_l) - \eta_l(x_l))^2 = \mathcal{O} \left( T^{-2k/(2k+1)} \right).$$

**Sketch of the proof:** Observe that

$$\begin{aligned}
\hat{\eta}_{l,T}(x_l) - \eta_l(x_l) &= \{\hat{\eta}_{l,T}(x_l) - \hat{\hat{\eta}}_{l,T}(x_l)\} + \{\hat{\alpha}_l(x_l) - E\hat{\alpha}_l(x_l)\} + \{E\hat{\alpha}_l(x_l) - \tilde{m}_{\psi,l}^T(x_l)\} \\
&\quad + E\{\hat{C}_T - C_{T,l} - C_l\}.
\end{aligned} \quad (8)$$

It follows that

$$\begin{aligned}
E\{\hat{\eta}_{l,T}(x_l) - \eta_l(x_l)\}^2 &\leq 4E\{\hat{\eta}_{l,T}(x_l) - \hat{\hat{\eta}}_{l,T}(x_l)\}^2 + 4E\{\hat{\alpha}_l(x_l) - E\hat{\alpha}_l(x_l)\}^2 + 4\{E\hat{\alpha}_l(x_l) - \tilde{m}_{\psi,l}^T(x_l)\}^2 \\
&\quad + 4E^2\{\hat{C}_T - C_{T,l} - C_l\}.
\end{aligned}$$

To prove the Theorem 1, it suffices to establish the following statements

$$E(\hat{\eta}_{l,T}(x_l) - \hat{\hat{\eta}}_{l,T}(x_l))^2 = \mathcal{O}\left(T^{-2k/(2k+1)}\right), \quad (9)$$

$$\text{Var}(\hat{\alpha}_l(x_l)) = \mathcal{O}\left(T^{-2k/(2k+1)}\right), \quad (10)$$

$$E\hat{\alpha}_l(x_l) - \tilde{m}_{\psi,l}^T(x_l) = \mathcal{O}\left(T^{-k/(2k+1)}\right), \quad (11)$$

$$E(\hat{C}_T - C_{T,l} + C_l) = \mathcal{O}\left(T^{-k/(2k+1)}\right). \quad (12)$$

*Proof of 9:* By combining the definitions of  $\hat{\eta}_{l,T}$  and  $\hat{\hat{\eta}}_{l,T}$  and the result of the lemma 1, we easily obtain, under the conditions on the kernel, the statement(9).

*Proof of 10:* Set  $\phi(t, s) = \text{Cov}\left(\frac{\tilde{Y}_{\psi,T,t}}{f_1(X_{t,1})h_{1,T}}K_1\left(\frac{x_1 - X_{t,1}}{h_{1,T}}\right), \frac{\tilde{Y}_{\psi,T,s}}{f_1(X_{s,1})h_{1,T}}K_1\left(\frac{x_1 - X_{s,1}}{h_{1,T}}\right)\right)$  and  $S_{a(T)} = \{(s, t) \in \mathbb{R}^2; |t - s| \leq a(T)\}$ , where  $a(T) = h_T^{-1}$ . We use the following decomposition

$$\text{Var}(\hat{\alpha}_1(x_1)) = \int_{[0,T]^2 \cap \Gamma} \phi(t, s) dt ds + \int_{[0,T]^2 \cap \Gamma^c \cap S_{a(T)}} \phi(t, s) dt ds + \int_{[0,T]^2 \cap \Gamma^c \cap S_{a(T)}^c} \phi(t, s) dt ds := A + E + F.$$

Under (C.1), (F.1), (K.1) – (K.2) and (Q.1), we have, for  $T$  large enough,

$$A = \mathcal{O}\left(1/Th_{1,T}\right) \quad \text{and} \quad E = \mathcal{O}\left(a(T)\|K_1\|_{L_1}^2 A_f(\Gamma)/T\right). \quad (13)$$

Using the Billingsley's inequality, it follows that

$$F = \mathcal{O}\left(1/Th_{1,T}^2 a(T)\right). \quad (14)$$

Combining (13) and (14), we obtain (10). To prove the statements (11) and (12), we use similar arguments as in the discrete case (see Camlong *et al.* (4)).

The next Theorem needs the following additional hypothesis.

(V)  $\liminf_{T \rightarrow \infty} Th_{l,T} \text{Var}(\hat{\eta}_{l,T}(x_l)) > 0$  where  $(\log(T)/T)^{k'/(2k'+d)} = o(h_{l,T}^k)$ .

**Theorem 2** *Under the hypotheses of Theorem 1 and (V) we have, for every  $\forall l \in [1, d]$  and  $\forall x_l \in \mathcal{C}_l$ ,*

$$\frac{\hat{\eta}_{l,T}(x_l) - \eta_l(x_l) - h_{l,T}^k b_l(x_l)}{\sqrt{\text{Var}(\hat{\eta}_{l,T}(x_l))}} \xrightarrow{\mathcal{L}} \mathcal{N}(0, 1).$$

**Sketch of the proof:** To obtain our theorem it suffices to show that

$$\sup_{x_l \in \mathcal{C}_l} |\hat{\eta}_{l,T}(x_l) - \hat{\hat{\eta}}_{l,T}(x_l)| = \mathcal{O}\left(\sup_{\mathbf{x} \in \mathcal{C}} |\hat{f}_T(\mathbf{x}) - f(\mathbf{x})|\right) \quad \text{a.s.}, \quad (15)$$

$$\frac{\{\hat{\alpha}_l(x_l) - E(\hat{\alpha}_l(x_l))\}}{\sqrt{\text{Var}(\hat{\alpha}_l(x_l))}} \longrightarrow \mathcal{N}(0, 1), \quad (16)$$

$$E\hat{\alpha}_l(x_l) - \tilde{m}_{\psi,l}^T(x_l) = \frac{(-h_{l,T})^k}{k!} m_l^{(k)}(x_l) \int_{\mathbb{R}} v_l^k K_l(v_l) dv_l + o(h_{l,T}^k), \quad (17)$$

$$\text{and} \quad E\{\hat{C}_T - C_{T,l} + C_l\} = \frac{h_{l,T}^k}{k!} \int_{\mathbb{R}} q_l^{(k)}(x_l) m_l(x_l) dx_l \int_{\mathbb{R}} v_l^k K_l(v_l) dv_l + o(h_{l,T}^k). \quad (18)$$

*Proof of 15:* The result arises directly from the definitions of estimates of  $\eta_l$  and the conditions on the kernels  $K_l, 1 \leq l \leq d$ .

*Proof of 16:* Set  $\frac{\{\hat{\alpha}_l(x_l) - E(\hat{\alpha}_l(x_l))\}}{\sqrt{\text{Var}(\hat{\alpha}_l(x_l))}} = \int_0^T Z_t dt =: S_T$ . We employ then the big block-small block procedure. Indeed setting,  $S_T = \sum_{j=1}^{k-1} (\nu_j + \xi_j) =: S'_T + S''_T$  where  $\nu_j = \int_{j(p+q)}^{j(p+q)+p} Z_t dt$  and  $\xi_j = \int_{j(p+q)+p}^{(j+1)(p+q)} Z_t dt$ . Now, it suffices to prove the following statements,

$$ES_T''^2 \rightarrow 0 \text{ as } T \rightarrow +\infty, \quad (19)$$

$$\left| E(e^{itS'_T}) - \prod_{j=0}^{k-1} E(e^{it\nu_j}) \right| \rightarrow 0 \text{ as } T \rightarrow +\infty, \quad (20)$$

$$\sum_{j=0}^{k-1} E[\nu_j^2] \rightarrow 1 \text{ as } T \rightarrow +\infty, \quad (21)$$

$$\text{and } \sum_{j=0}^{k-1} E[\nu_j^2 \mathbb{I}_{\{\nu_j^2 > \epsilon\}}] \rightarrow 0 \text{ as } T \rightarrow +\infty. \quad (22)$$

To show (21) et (22), we use the same arguments as those deployed in the discrete case.

**Lemma 2** *Under the conditions (C.1) – (C.4), (F.1) – (F.2), (K.1), (Q.1) – (Q.2) and (H.1) – (H.2), we have, for every  $1 \leq l \leq d$  and for any  $x_l \in \mathcal{C}_l$  and every  $(\alpha, \beta) \in [0; 0, 5[\times]0, 5; 1[$ ,*

$$\liminf_{T \rightarrow \infty} P\left(T^{\frac{k}{2k+1}} \{\hat{\eta}_{l,T}(x_l) - \eta_l(x_l) - h_{l,T}^k b_l(x_l)\} \in [Aq_\alpha; Aq_\beta]\right) \geq \beta - \alpha, \quad (23)$$

where  $A := (\limsup_{T \rightarrow +\infty} T^{\frac{2k}{2k+1}} \text{Var}(\hat{\eta}_{l,T}(x_l)))^{1/2}$  and  $q_u$  is such that  $P(\mathcal{N}(0, 1) < q_u) = u$ .

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